necessarily long proofs of theorems), are usually due to one of two phenomena: First, there is a true redundancy, whereby two phases of the computation (proof) merely cancel each other out. See [6] for the analysis of such a theorem proof. Secondly, the computation (proof) does more than is really needed, or wanted. See [7] for such a theorem proof. In our present case the inefficiency is clearly of the second type.

Finally, we note that D. H. Lehmer, in [4], showed how most of the subtractions in (10) could also be eliminated. It is not necessary to compute every γ_n/γ_{n+1}, but merely those at periodic intervals. It was his method that was used here in computing Table 1. Any interested reader will now have no difficulty in determining precisely how Lehmer manages to obtain this still greater efficiency.

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Distributions of Mersenne Divisors

By Sidney Kravitz

By driving computers to the limit of their capability, 23 prime Mersenne Numbers have been discovered [1]. The list of known divisors on the other hand is a large one. As a result of both of these lists, conjectures have appeared regarding the expected number of primes and of divisors [1], [2], [3]. This note presents additional data relative to the observed frequency of divisors of Mersenne Numbers.

Each divisor, q, of the Mersenne Number $M_p = 2^p - 1$, p a prime, is of the form $2kp + 1$ and of the form $8L \pm 1$. (Therefore $k \neq 4n + 2$.) Thus if k is known for a particular p, it identifies the divisor. The divisors of the Mersenne Numbers, $3 \leqq p < 100{,}000$ have been examined for $k \leqq 200$. The frequency f, with which the various values of k occur is given in Table 1. This table shows that the frequency of k tends to decrease as k increases, but those k with a large number of small divisors, e.g. 12, 24, and 60, occur with much greater frequency than their neighbors on the list.

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TABLE 1

k	f	k	f	k	f	k	f	k	f	k	f	k	f
1	581	33	32	65	7	97	7	129	12	161	5	193	2
3	350	35	13	67	8	99	3	131	6	163	5	195	6
4	266	36	41	68	12	100	12	132	11	164	5	196	6
5	141	37	17	69	10	101	4	133	2	165	7	197	1
7	84	39	29	71	8	103	4	135	6	167	3	199	0
8	122	40	25	72	28	104	9	136	4	168	10	200	7
9	115	41	11	73	5	105	10	137	5	169	3		
11	61	43	10	75	10	107	3	139	2	171	10		
12	162	44	19	76	10	108	17	140	6	172	3		
13	47	45	24	77	7	109	4	141	9	173	2		
15	88	47	11	79	6	111	8	143	1	175	2		
16	56	48	39	80	20	112	10	144	15	176	7		
17	23	49	6	81	11	113	8	145	4	177	6		
19	26	51	16	83	6	115	6	147	3	179	1		
20	56	52	15	84	27	116	9	148	5	180	7		
21	47	53	8	85	5	117	6	149	2	181	2		
23	18	55	14	87	9	119	5	151	5	183	6		
24	68	56	30	88	16	120	13	152	5	184	4		
25	34	57	15	89	8	121	4	153	7	185	1		
27	27	59	7	91	5	123	9	155	4	187	2		
28	37	60	47	92	12	124	7	156	13	188	1		
29	11	61	5	93	6	125	6	157	2	189	8		
31	16	63	17	95	5	127	2	159	2	191	1		
32	24	64	17	96	16	128	8	160	5	192	13		

This writer advocates that the k values for each division be listed in future tables of divisors rather than the divisor itself, inasmuch as the k reveals more about the character of the divisor [4], [5], [6], [7], [8].

The referee points out that well-known heuristic arguments, cf. [3, Example 39S, p. 214], would suggest that $f(1)$ is asymptotic, but rather slowly, and from below, to one-half the number of twin primes up to the same limit. Further, cf. [3, Example 16, pp. 29, 169], one expects that $f(3)$ is asymptotic, again slowly from below, to $\frac{2}{3}$ of $f(1)$. Comparison of the data here with the 1224 twins up to 100,000 is satisfactory.

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